Structural Analysis of Tapered Antenna Elements

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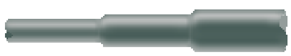
Introduction



tapered element

Long, horizontal antenna elements or vertical antennas require a sound mechanical design. This requirement is not merely for the sake of safety; it equally provides reliability and convenience. After all, it is no fun having to miss your antenna after a heavy storm or freezy blizzard went by. This calculator applies the cantilever method to perform the structural analysis of a horizontal or vertical circular cylindrical or square stock antenna structure, consisting of up to eight sections with eight additional overlaps. Ice, wind and gravitational loads are accounted for.

Keep it safe, have fun and hopefully you will sleep better now during those stormy nights!

The *Tapering to Perfection* tutorial by the late L. B. Cebik, W4RNL (SK), deals with the electrical modelling of tapered antenna elements.

Legal disclaimer

See sections 15, 16 & 17 of the GNU GPL v3.

Metric tube dimensions

Table 1: Common metric tube dimensions (in mm)

OD	t_{wall}	*ID*		*OD*	t_{wall}	*ID*		*OD*	t_{wall}	*ID*
4	1	2		30	5	20		52	1.5	49
6	1	4		30	3	24		55	2.5	50
8	1	6		30	2.5	25		57	2	53
10	1	8		30	2	26		60	5	50
12	1	10		30	1.5	27		60	3	54
13	1	11		32	1.5	29		60	1.5	57
15	1.5	12		35	5	25		62	2	58
16	1.5	13		35	2.5	30		70	5	60
18	1.5	15		35	2	31		70	3	64
19	1.5	16		36	1.5	33		80	5	70
20	5	10		38	4	30		80	4	72
20	2	16		40	5	30		80	2	76
20	1.5	17		40	2.5	35		90	5	80
22	2	18		40	2	36		100	5	90
22	1.5	19		40	1.5	37		100	2	96
25	5	15		42	3	36		110	5	100
25	2.5	20		45	2.5	40		120	5	110
25	2	21		45	2	41		150	5	140
25	1.5	22		48	1.5	45		160	5	150
28	1.5	25		50	5	40		200	5	190
				50	3	44				
				50	2.5	45				
				50	2	46				
				50	1.5	47				

Metric rod dimensions

Rods are commonly available in following metric diameters:[16] 6, 7, 8, 10, 12, 15, 16, 20, 25, 28, 30, 35, 40 and 50 mm. Rods are modelled by entering a wall thickness equal to half the rod diameter.

Factor of safety

The calculator employs a factor of safety (FS) as proposed by Ullman.[17] This safety factor corresponds to the product of a number of contributing cofactors:

$$FS = \frac{F_{\text{max}}}{F_i} = FS_{\text{material}} \cdot FS_{\text{stress}} \cdot FS_{\text{geometry}} \cdot FS_{\text{failure analysis}} \cdot FS_{\text{reliability}} \quad (1)$$

Following respective values apply to this calculator. Please, refer to the tables below for a detailed justification.

$$FS = (1.1) \cdot (1.2) \cdot (1.0) \cdot (1.0) \cdot (1.25) = 1.65 \quad (2)$$

$$\frac{F_i}{F_{\text{max}}} = \frac{1}{FS} = \frac{1}{1.65} = 0.6060... \approx 60\% \quad (3)$$

The calculator notifies the user when the applied load transgresses 60% of the maximum allowable shear force or bending moment. This is indicated by an orange or red background, where red signifies guaranteed failure with at least permanent deformation. Applying a conservative safety factor will also result in a reduced horizontal element sag.

Table 2: Estimating the contribution for the material

FS_{material}	situation
1.0	If the properties for the material are well known, if they have been experimentally obtained from tests on a known to be identical to the component being designed and from tests representing the loading to be applied
1.1	If the material properties are known from a handbook or are manufacturer's values
1.2–1.4	If the material properties are not well known

Table 3: Estimating the contribution for the load stress

FS_{stress}	situation
1.0–1.1	If the load is well defined as static or fluctuating, if there are no anticipated overloads or shock loads, and if an accurate method of analyzing the stress has been used
1.2–1.3	If the nature of the load is defined in an average manner, with overloads of 20–50%, and the stress analysis method may result in errors less than 50%
1.4–1.7	If the load is not well known or the stress analysis method is of doubtful accuracy

Table 4: Estimating the contribution for geometry (unit-to-unit)

FS_{geometry}	situation
1.0	If the manufacturing tolerances are tight and held well
1.0	If the manufacturing tolerances are average
1.1–1.2	If the dimensions are not closely held

Table 5: Estimating the contribution for failure analysis

$FS_{\text{failure theory}}$	situation
1.0–1.1	If the failure analysis to be used is derived for the state of stress, as for uniaxial or multiaxial static stresses, or fully reversed uniaxial fatigue stresses
1.2	If the failure analysis to be used is a simple extension of the preceding theories, such as for multiaxial, fully reversed fatigue stresses or uniaxial nonzero mean fatigue stresses
1.3–1.5	If the failure analysis is not well developed, as with cumulative damage or multiaxial nonzero mean fatigue stresses

Table 6: Estimating the contribution for reliability

$FS_{\text{reliability}}$	situation
1.1	If the reliability of the part needs not be high, for instance, less than 90%
1.2–1.3	If the reliability of tha part must have an average of 92–98%
1.4–1.6	If the reliability must be high, say, greater than 99%

Formulas

Wind pressure

The peak wind speed v is converted to a peak wind pressure p:

$$p_{\text{wind}} = \frac{\rho_{\text{air},T}\, v_{\text{wind}}^2}{2}\, C_d \tag{4}$$

where:

$\rho_{\text{air},\,T}$ = 1.3413 kg/m³ : the volumetric mass density of air at -10 °C

C_d : the drag coefficient at subcritical Reynolds numbers; 1.18 for long circular cylindrical sections, and maximum 2.05 for long square sections[1,2]

Geometry

$$ID = OD - 2\, t_{\text{wall}} \tag{5}$$

The cross-sectional areas A of circular, respectively square, hollow tube is given by:

$$A_{\circledcirc} = \frac{\pi}{4}\left(OD^2 - ID^2\right) \quad \text{and} \quad A_{\boxdot} = OD^2 - ID^2 \tag{6}$$

The elastic section modulus S of a circular, respectively, square hollow section is given by:

$$S_{\circledcirc} = \frac{\pi}{32} \frac{OD^4 - ID^4}{OD} \quad \text{and} \quad S_{\boxdot} = \frac{1}{6} \frac{OD^4 - ID^4}{OD} \tag{7}$$

Maxima

The maximum allowable shear force F_{max} and bending moment M_{max} are, respectively:

$$F_{\text{max}} = YS \cdot A \quad \text{and} \quad M_{\text{max}} = YS \cdot S \tag{8}$$

Above these values, the material will start to fail with permanent deformations and bending. Excessive transgressions will cause the section to simply break.

Distributed loads

The mass and uniformly distributed load of the material are, respectively:

$$m_{\text{material}} = V \cdot \rho = A \cdot \ell \cdot \rho \quad \text{and} \quad q_{\text{material}} = \frac{10\, m_{\text{material}}}{\ell} \tag{9}$$

Likewise, the cross-sectional area A_{ice} of a circular, respectively square, hollow ice section is given by:

$$A_{\circledcirc\,\text{ice}} = \frac{\pi}{4}\left[(OD + 2\,t_{\text{ice}})^2 - OD^2\right] \quad \text{and} \quad A_{\boxdot\,\text{ice}} = (OD + 2\,t_{\text{ice}})^2 - ID^2 \tag{10}$$

and therefore:

$$m_{\text{ice}} = V \cdot \rho = A \cdot \ell \cdot \rho \quad \text{and} \quad q_{\text{ice}} = \frac{10\, m_{\text{ice}}}{\ell} \tag{11}$$

The uniformly distributed wind load is calculated from the projected area and the wind pressure:

$$q_{\text{wind}} = \frac{A_{\text{projected}}\, p_{\text{wind}}}{\ell} = (OD + 2\,t_{\text{ice}})\, p_{\text{wind}} \tag{12}$$

Resulting forces & moments

The magnitudes of the shear force F_i, respectively, bending moment M_i at the end of section i are given by:

$$F_i = F_{i-1} + F_{0,i} + q_i\,\ell \quad \text{and} \quad M_i = M_{i-1} + M_{0,i} + (F_{i-1} + F_{0,i})\,\ell + \frac{q_i\,\ell^2}{2} \tag{13}$$

For the sake of simplicity, only shear & point forces, bending moments and the uniformly distributed load are indexed. However, all other quantities are equally specific to section i.

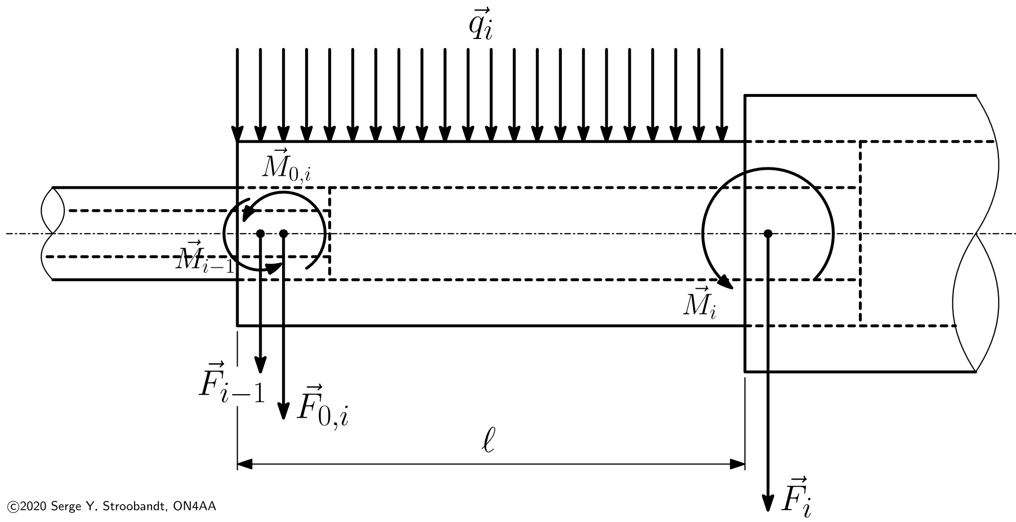


Figure 1: Schematic representation of the shear force F_{i-1}, point force $F_{0,\,i}$, bending moments M_{i-1} & $M_{0,\,i}$ and uniformly distributed load q_i acting upon the beginning of section i and over the section length ℓ. The resulting shear force F_i and bending moment M_i at the end of the section are also shown. Note that the two regions where the tubing overlaps, could equally be modelled as seperate sections.

Brython source code

Here is the Brython code of this calculator. Brython code is not intended for running stand alone, even though it looks almost identical to Python 3. Brython code runs on the client side in the browser, where it is transcoded to secure Javascript.

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Download: `tapers.py`

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