The Effective Diameter of a Single-Layer Helical Coil Inductor as a Function of the Proximity Factor

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Abstract

Inductor calculators typically employ the mean and inner physical diameters (respectively D and $D-d$) of a single-layer helical coil with wire diameter d to bracket the inductance of the coil between two widely-spaced theoretical limits. However, it has often been hinted that the actual effective diameter D_{eff} of a coil might be linked to the proximity factor Φ. In order to do away with the ambiguity of an inductance range result, a new formula is deduced in which the effective diameter D_{eff} is a mere function of the proximity factor Φ. As a result of this effort, an inductance calculator will return only a single value of inductance.

1 Introduction

Inductor calculators typically employ the mean and inner physical diameters (respectively: D and $D-d$) of a single-layer helical coil with wire diameter d to bracket the inductance of the coil between two widely-spaced theoretical limits[1]. However, it has often been alluded that the actual effective diameter D_{eff} of a coil aught to be linked to the proximity factor Φ. The latter factor is used for calculating the AC resistance of the coil and is readily obtained by interpolation from Medhurst's table of experimental data[1, 2]. In order to do away with the ambiguity of an inductance range result, at continuation, a new formula will be deduced in which the effective diameter D_{eff} is a mere function of the proximity factor Φ. As a result of this trivial effort, it has become possible to develop an inductance calculator that will return only a single value of inductance.

*This text and containing formula would not have come into existence without David Knight's, G3YNH, excellent overview[1] that should be read as a general introduction to this topic.

2 Deduction of D_{eff} as a function of Φ

The proximity factor Φ *is defined as the ratio of total AC resistance including the contribution of the proximity effect over AC resistance without the contribution of the proximity effect.*

$$\boxed{\Phi \equiv \frac{R_{\text{ac},\Phi}}{R_{\text{ac}}}}$$

Since resistance is inversely proportional to the diameter of the current-carrying conductor cross-section (see Figure 1); $R_{\text{ac}} \propto \frac{1}{d^2}$ and $R_{\text{ac},\Phi} \propto \frac{1}{d_\Phi^2}$:

$$\begin{aligned} \Rightarrow \quad & \Phi = \frac{d^2}{d_\Phi^2} \\ \Rightarrow \quad & d_\Phi = \sqrt{\frac{d^2}{\Phi}} = \frac{d}{\sqrt{\Phi}} \end{aligned} \tag{1}$$

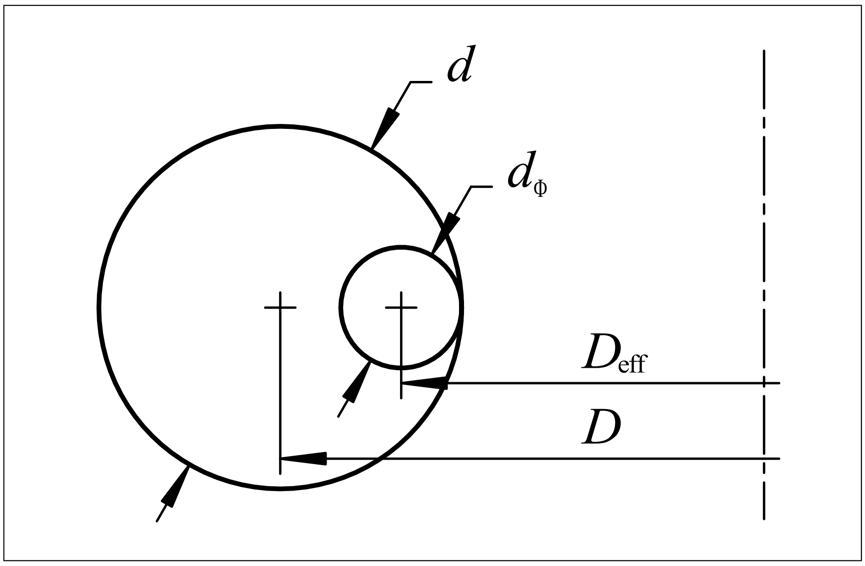

Figure 1: Quenching of the current-carrying conductor cross-section under the proximity effect and leading to a reduction of the effective diameter of the coil

With reference to Figure 1:

$$\begin{aligned} D_{\text{eff}} &= D - 2\left(\frac{d}{2} - \frac{d_\Phi}{2}\right) \\ &= D - d + d_\Phi \\ &= D - d + \frac{d}{\sqrt{\Phi}} \qquad \text{subst. (1)} \end{aligned}$$

$$\boxed{D_{\text{eff}} = D - d\left(1 - \frac{1}{\sqrt{\Phi}}\right)} \tag{2}$$

3 Application

The effective diameter D_{eff} obtained from (2) is used to calculate the inductance of the single-layer helical coil:

$$L = \mu\pi \frac{D_{\text{eff}}^2}{4} \frac{N^2}{\ell} k_L - \mu \frac{D_{\text{eff}}}{2} N \left(k_s + k_m\right) \tag{3}$$

where:

μ: permeability of the coil core
N: number of turns
ℓ: length of the coil, measured from the connecting wires centre to centre
k_L: field non-uniformity correction factor according to Lundin[3]
k_s: round wire self-inductance correction factor according to Rosa[4, 5]
k_m: round wire mutual-inductance correction factor according to Grover and Knight[1, 6]

4 Limitations

Although being more determined than effective diameter bracketing, inductance calculations based on this formula remain approximative for two reasons:

1. As is the case with diameter bracketing, this formula equally assumes that current under the proximity effect will quench while retaining a circular transversal distribution. This is most probably not the case.

2. As with diameter bracketing, the formula does not take into account frequency-dependent disturbances of the transversal current distribution, such as the skin effect.

These transversal current distribution disturbances which are not accounted for, may move the current centre further inward thereby reducing the effective diameter and the resulting inductance even more. The inductance obtained through (3) will therefore be slightly overstated. Nonetheless, the result will be in most cases more accurate and certainly less ambiguous than the theoretical extremes (or average of these) given by diameter bracketing.

References

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[4] Rosa E. B.: *Bulletin of the Bureau of Standards,* Vol. 2, 1906, pp. 161-187.

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